





# Functions

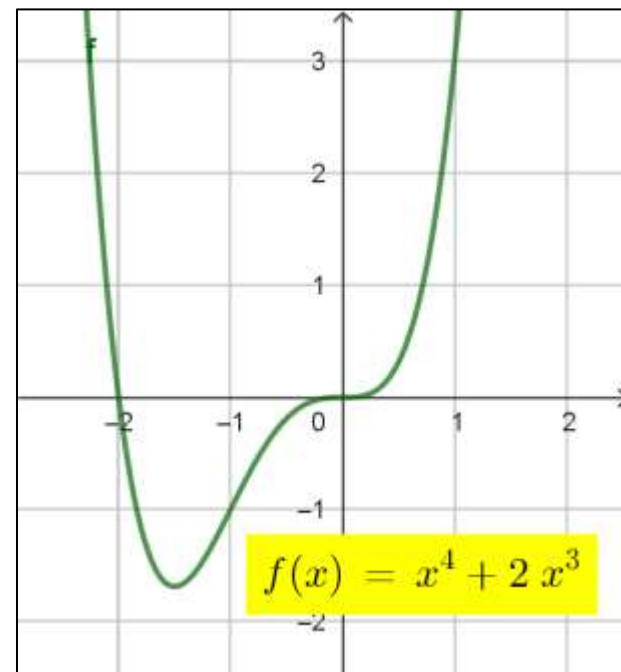
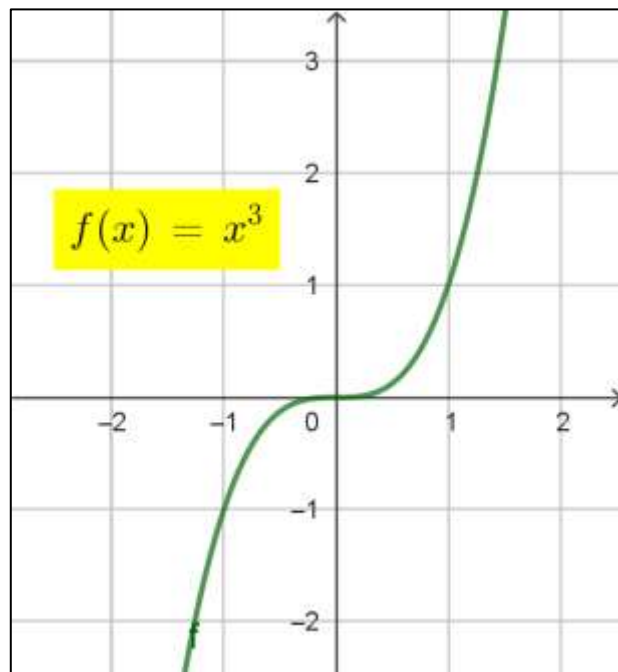
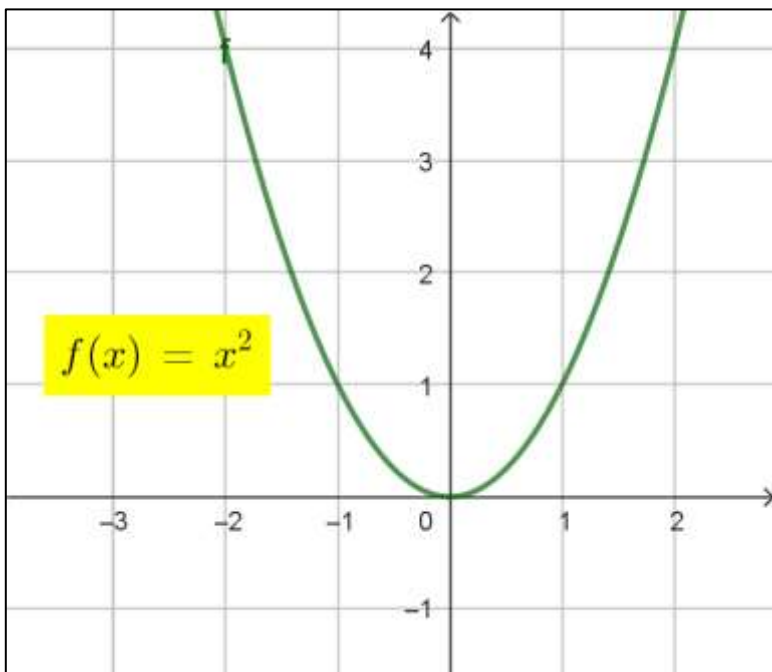
Graph of a function



# Graph of a function

We've learned that the curve is a form of representing a function  $f$ . It is the set of points of coordinates  $(x; f(x))$  where  $x \in D_f$ .

Example:



# How to plot the graph of a function?

Example 1:

Consider the function  $f$  defined over  $\mathbb{R}=]-\infty;+\infty[$  by  $f(x) = x^2$ . Study the variations of  $f$  and plot its curve ( $C_f$ ).

1. Limits at the endpoints:

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} x^2 = +\infty \quad ; \quad \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} x^2 = +\infty$$

2. Derivative + zeroes of  $f'(x)$ :

$$f'(x) = 2x$$

$$f'(x) = 0 \quad ; \quad 2x = 0 \quad ; \quad x = \frac{0}{2} = 0 \quad y = 0^2 = 0$$



# How to plot the graph of a function?

Example 1:

3. Table of variations.

$x$	$-\infty$	$0$	$+\infty$
$f'(x)$	$-$	$0$	$+$
$f(x)$	$+\infty$	$0$	$+\infty$

Diagram showing the variation of  $f(x)$  from  $+\infty$  at  $-\infty$  to  $0$  at  $0$ , and then increasing to  $+\infty$  at  $+\infty$ .

4. Particular points if they exist

The particular points are the intersecting points with axes of coordinates:

$(x'x)$ : for  $y = 0$  ;  $x^2 = 0$  ;  $x = 0$

$(y'y)$ : for  $x = 0$  ;  $y = 0$

So 1 p.p. (0;0)



# How to plot the graph of a function?

Example 1:

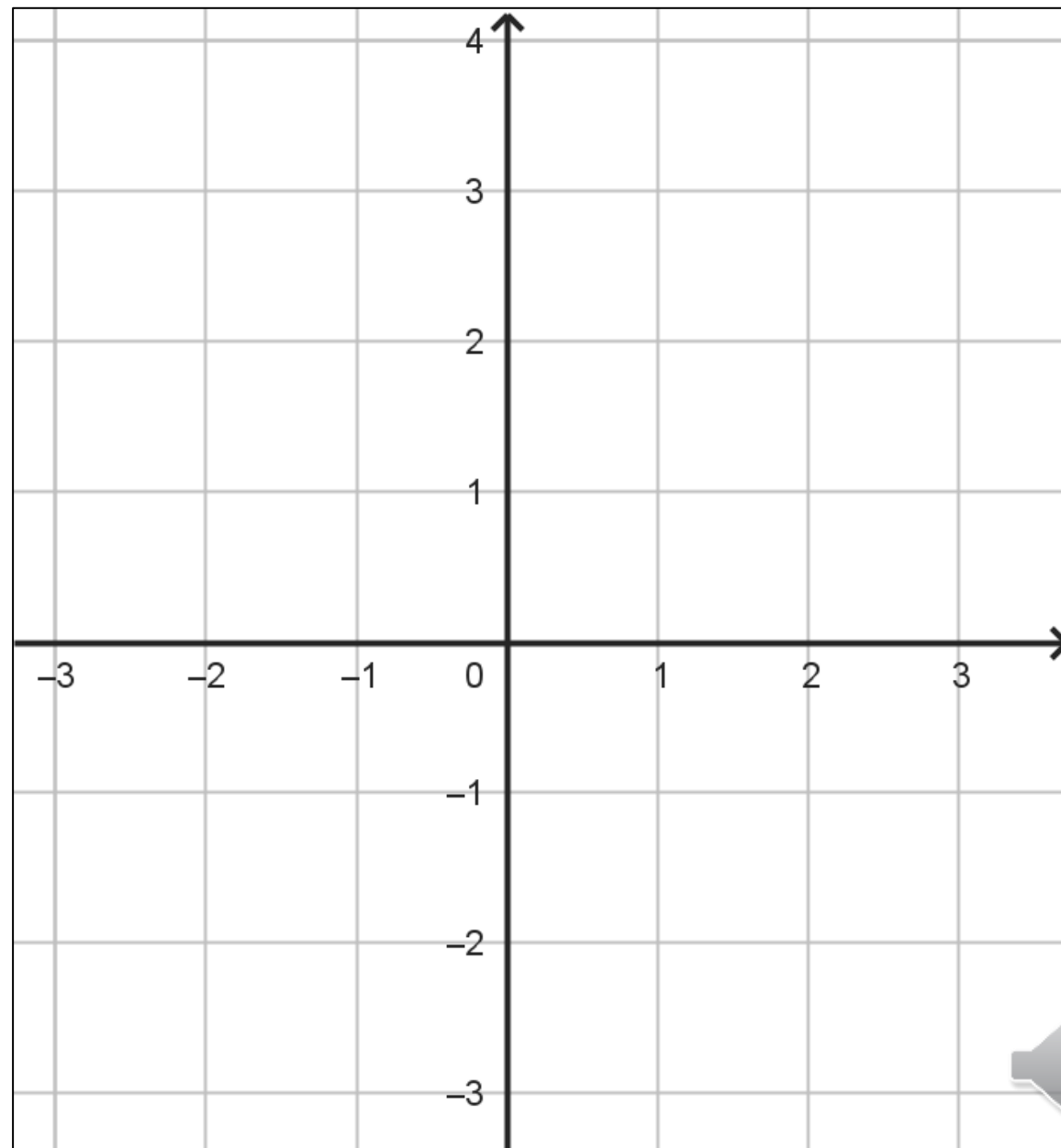
5. Plot the curve

Note that:

The curve of  $f$  is a reflection to the table of variations of  $f$ .

$x$	$-\infty$	$0$	$+\infty$
$f'(x)$	$-$	$0$	$+$
$f(x)$	$+\infty$	$0$	$+\infty$

Diagram illustrating the reflection of the function values from the table of variations to the graph. Arrows point from the  $+\infty$  values in the  $f(x)$  row to the  $0$  value in the  $f(x)$  row, indicating a reflection across the x-axis.



# How to plot the graph of a function?

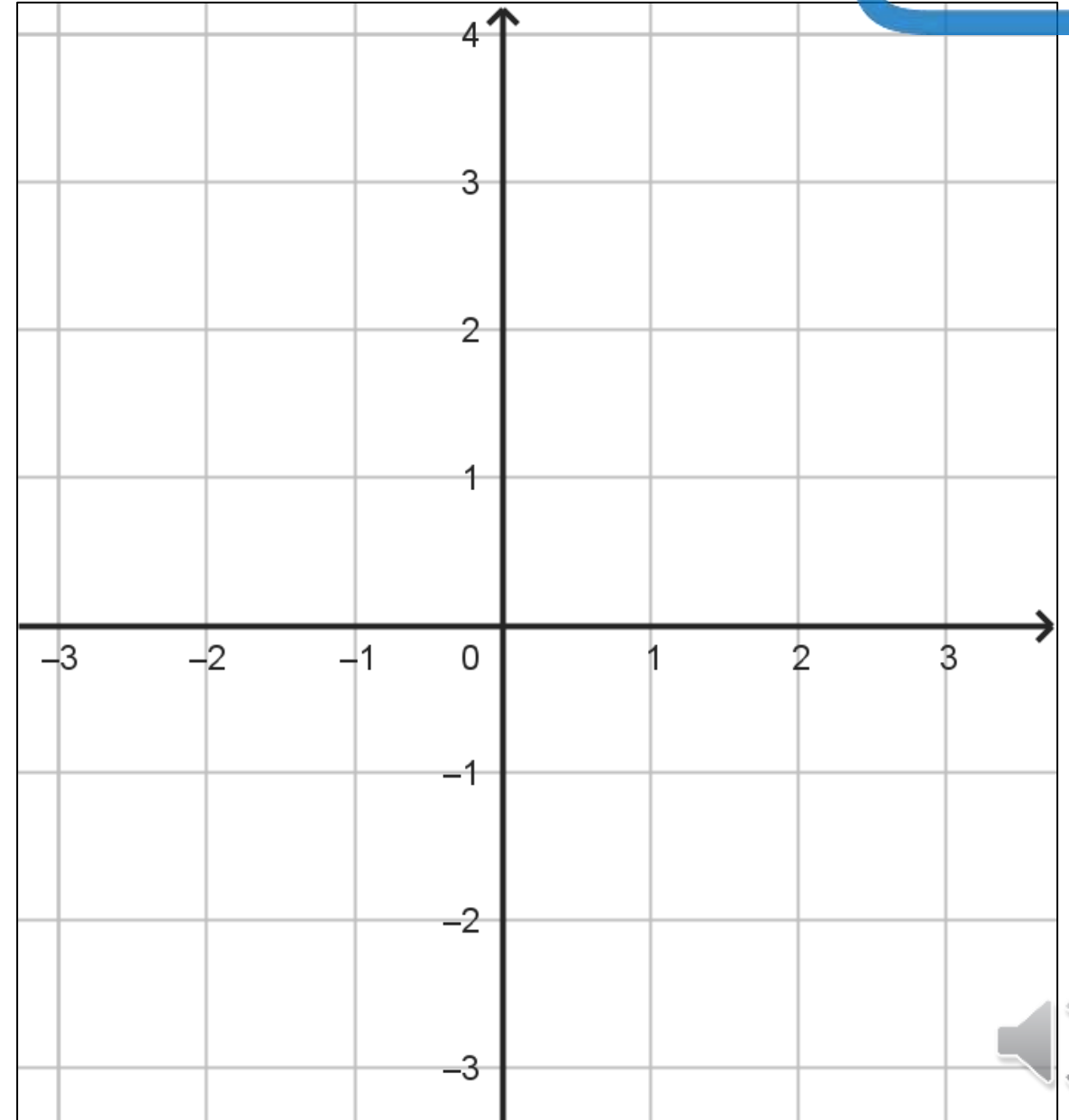
Example 1:

5. Plot the curve

Note that:

The curve of  $f$  is a reflection to the table of variations of  $f$ .

$x$	$-\infty$	$0$	$+\infty$
$f'(x)$	$-$	$0$	$+$
$f(x)$	$+\infty$	$0$	$+\infty$



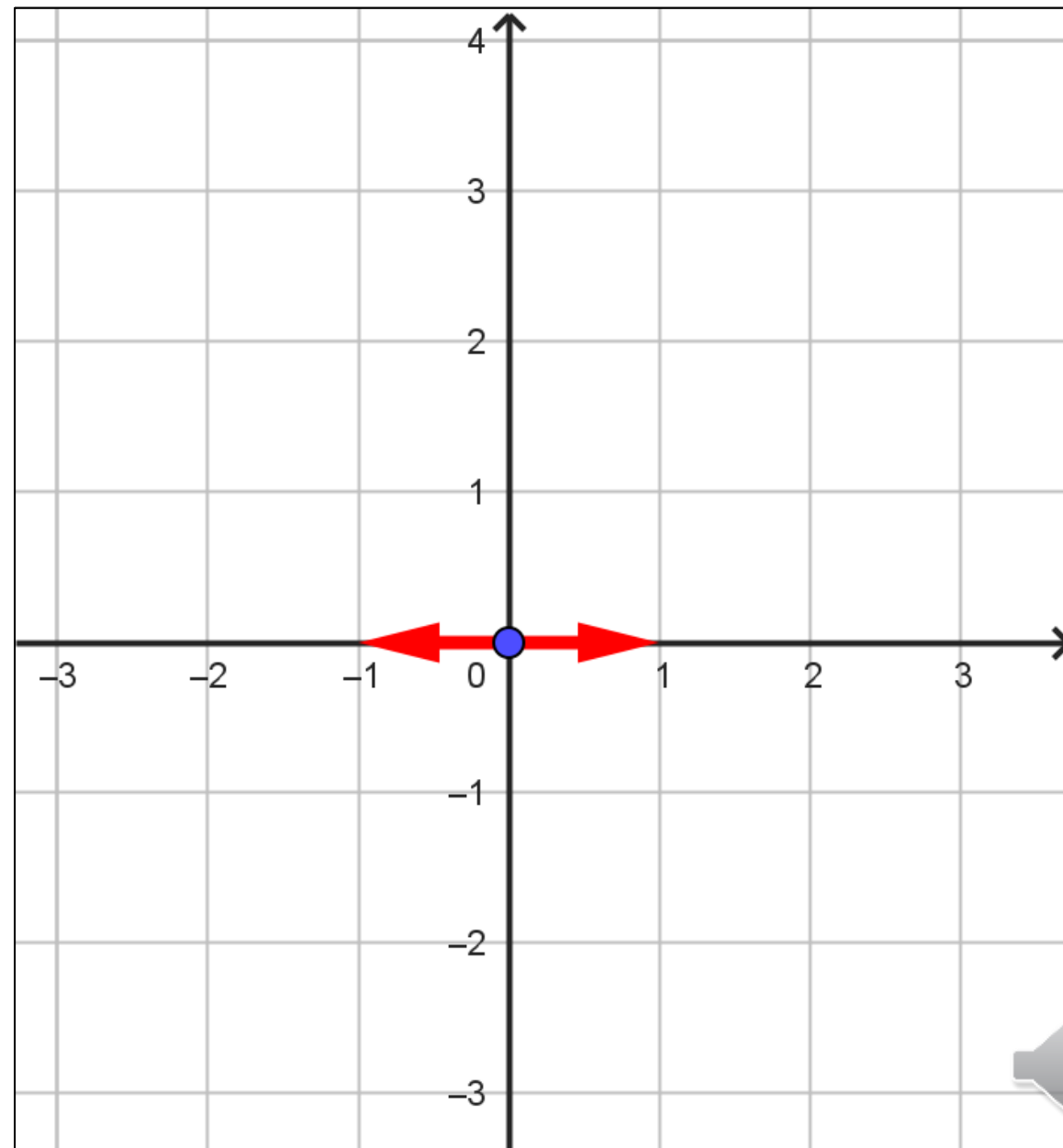
# How to plot the graph of a function?

Example 1:

5. Plot the curve

$x$	$-\infty$	0	$+\infty$
$f'(x)$	-	0	+
$f(x)$	$+\infty$	0	$+\infty$

After plotting the extrema, plot the particular points. But in this case the particular is same the extremum





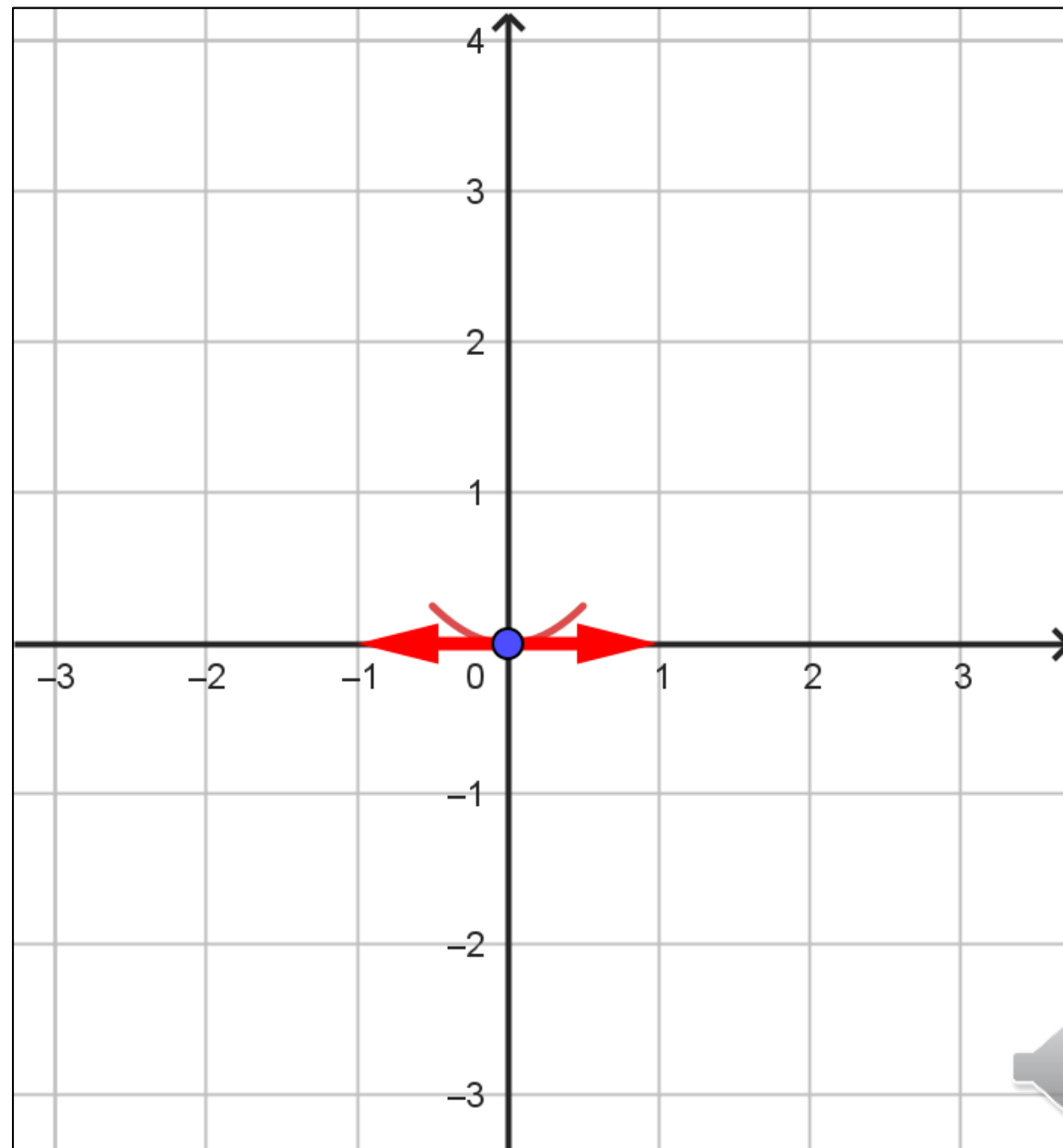
# How to plot the graph of a function?

Example 1:

5. Plot the curve

$x$	$-\infty$	$0$	$+\infty$
$f'(x)$	$-$	$0$	$+$
$f(x)$	$+\infty$	$0$	$+\infty$

Note that at the local extremum the curve must not be sharp.



# How to plot the graph of a function?

Example 1:

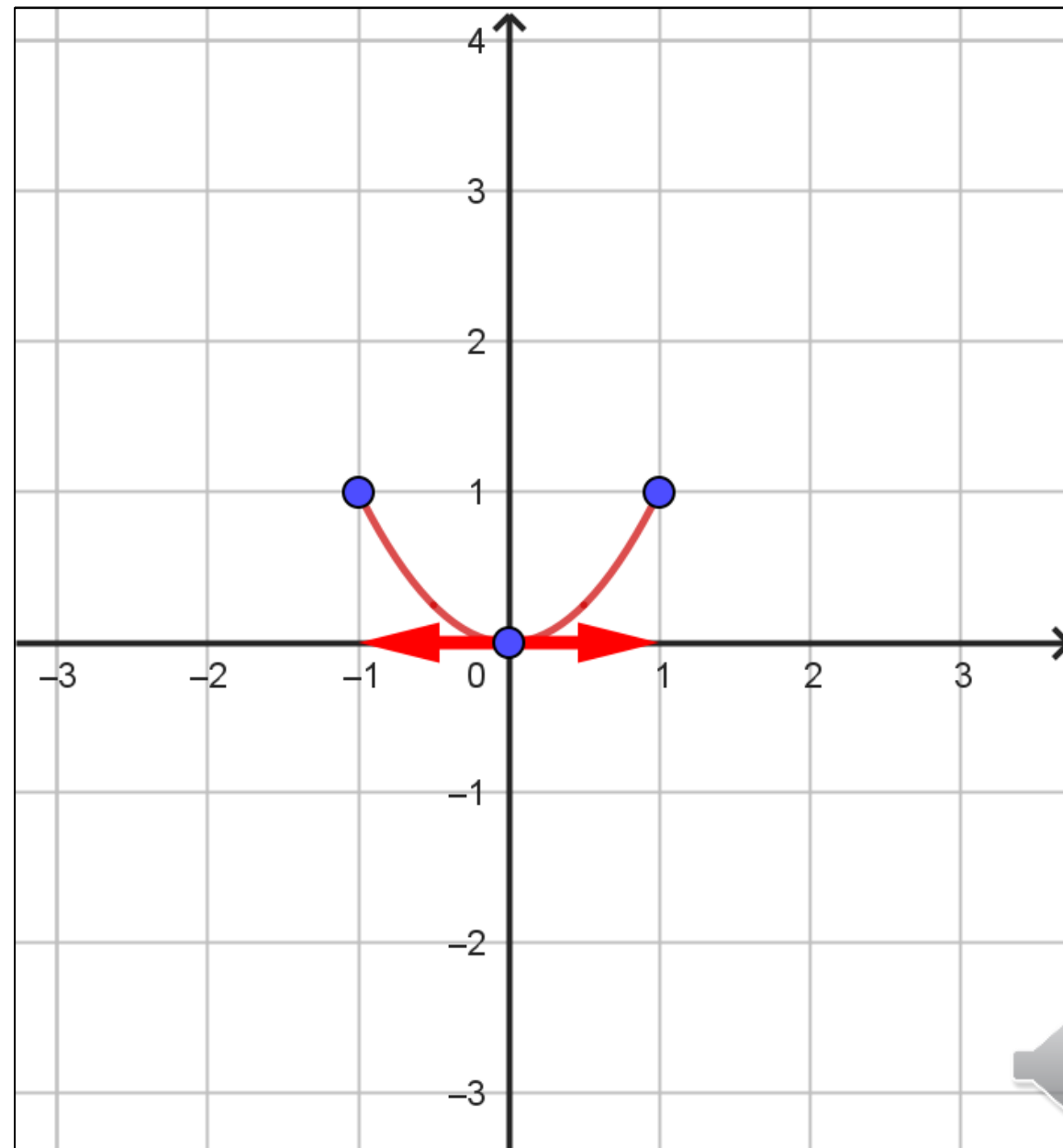
5. Plot the curve

$x$	$-\infty$	$0$	$+\infty$
$f'(x)$	$-$	$0$	$+$
$f(x)$	$+\infty$	$0$	$+\infty$

To help in drawing, we can find some helping points:

$$\text{for } x = -1 ; y = (-1)^2 = 1$$

$$\text{for } x = 1 ; y = (1)^2 = 1$$



# How to plot the graph of a function?

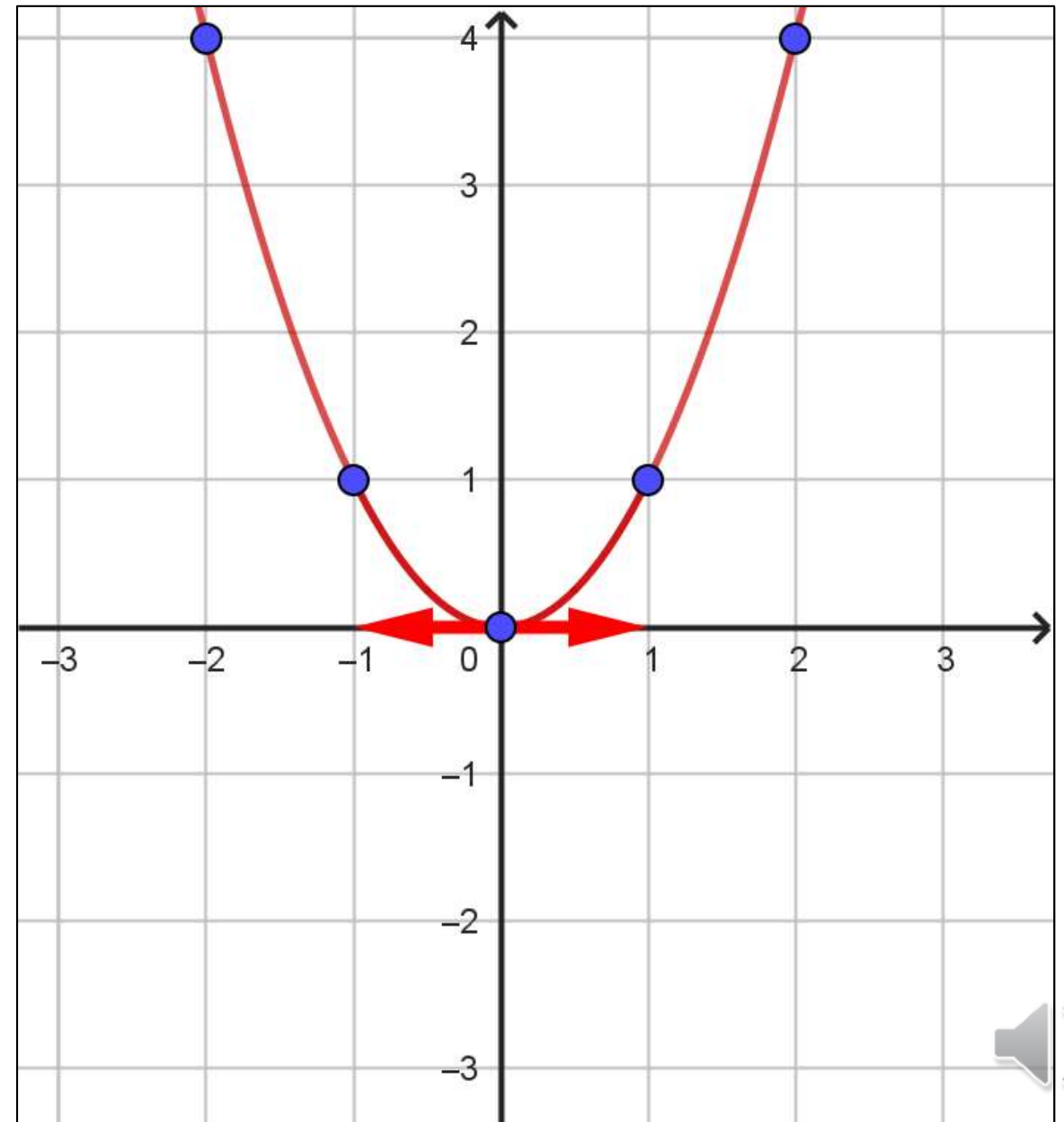
Example 1:

5. Plot the curve

$x$	$-\infty$	$0$	$+\infty$
$f'(x)$		$- \longleftrightarrow 0 \rightarrow +$	
$f(x)$	$+\infty$	$0$	$+\infty$

Remark:

Don't stop at a point, draw a part of the curve after the point to represent infinity branches.



# How to plot the graph of a function?

Example 2:

Consider the function  $f$  defined over  $\mathbb{R}=]-\infty;+\infty[$  by  $f(x) = x^3 - 3x^2 + 3$ .

Study the variations of  $f$  and plot its curve ( $C_f$ ).

1. Limits at the endpoints:

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} x^3 = +\infty \quad ; \quad \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} x^3 = -\infty$$

2. Derivative + zeroes of  $f'(x)$ :

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \quad ; \quad 3x^2 - 6x = 0 \quad ; \quad 3x(x - 2) = 0 \quad ; \quad x = 0 \text{ or } x = 2$$
$$y = 3 \quad y = -1$$



# How to plot the graph of a function?

Example 2:

3. Table of variations.

$x$	$-\infty$	$0$	$2$	$+\infty$	
$f'(x)$	$+$	$0$	$-$	$0$	$+$
$f(x)$	$-\infty$	$3$	$-1$	$+\infty$	

4. Particular points if they exist

$(x'x)$ : for  $y = 0$  ;  $x^3 - 3x^2 + 3 = 0$

*Using calculator:*

$x \approx -0.9$  ;  $x \approx 1.3$  ;  $x \approx 2.5$

$(y'y)$ : for  $x = 0$  ;  $y = 3$

So 4 p.p.:

$(0;3)$  ;  $(-0.9;0)$  ;  $(1.3;0)$  ;  $(2.5 ; 0)$

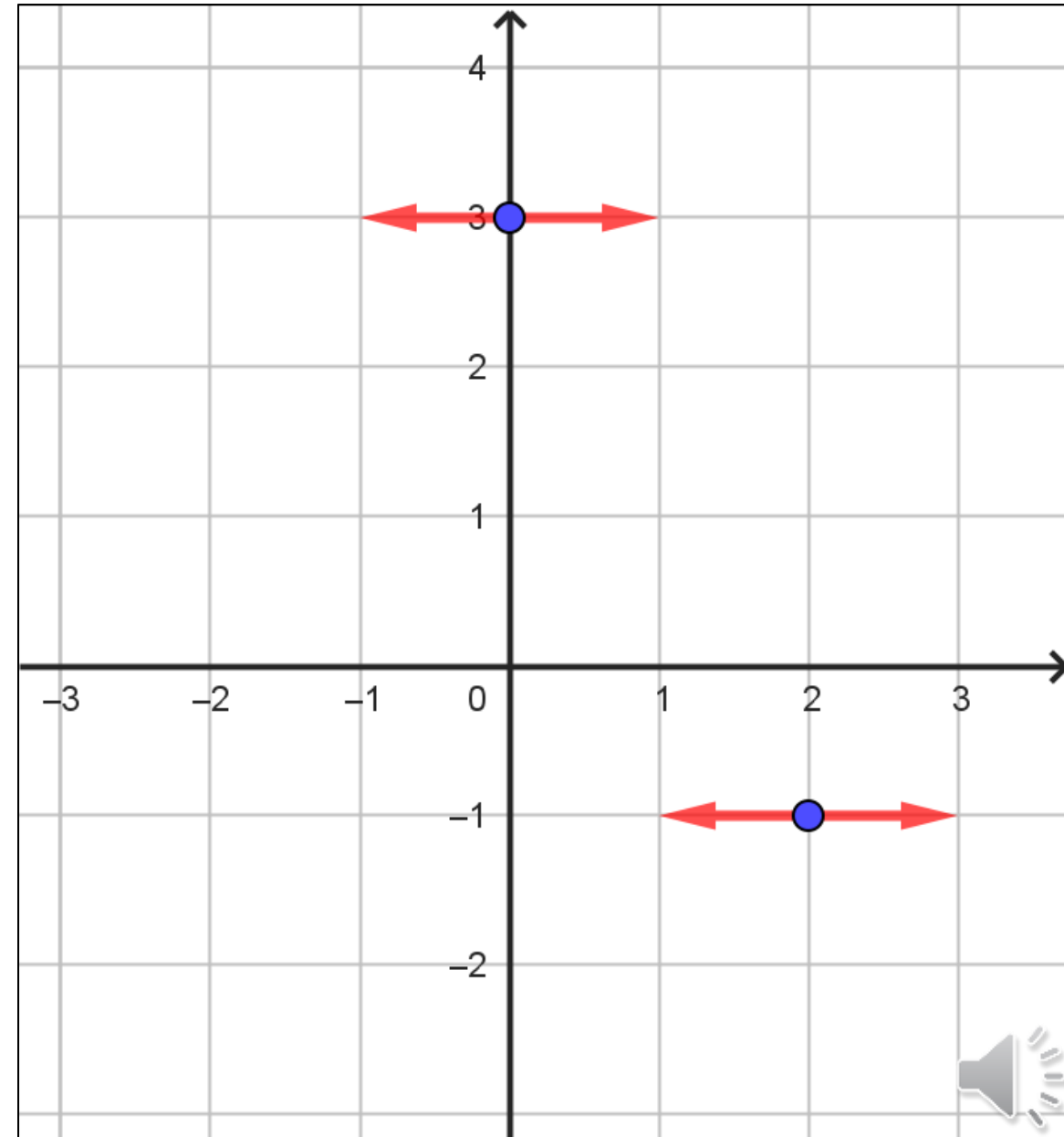


# How to plot the graph of a function?

Example 1:

5. Plot the curve

$x$	$-\infty$	0	2	$+\infty$
$f'(x)$	+	0	-	+
$f(x)$	$-\infty$	3	-1	$+\infty$



# How to plot the graph of a function?

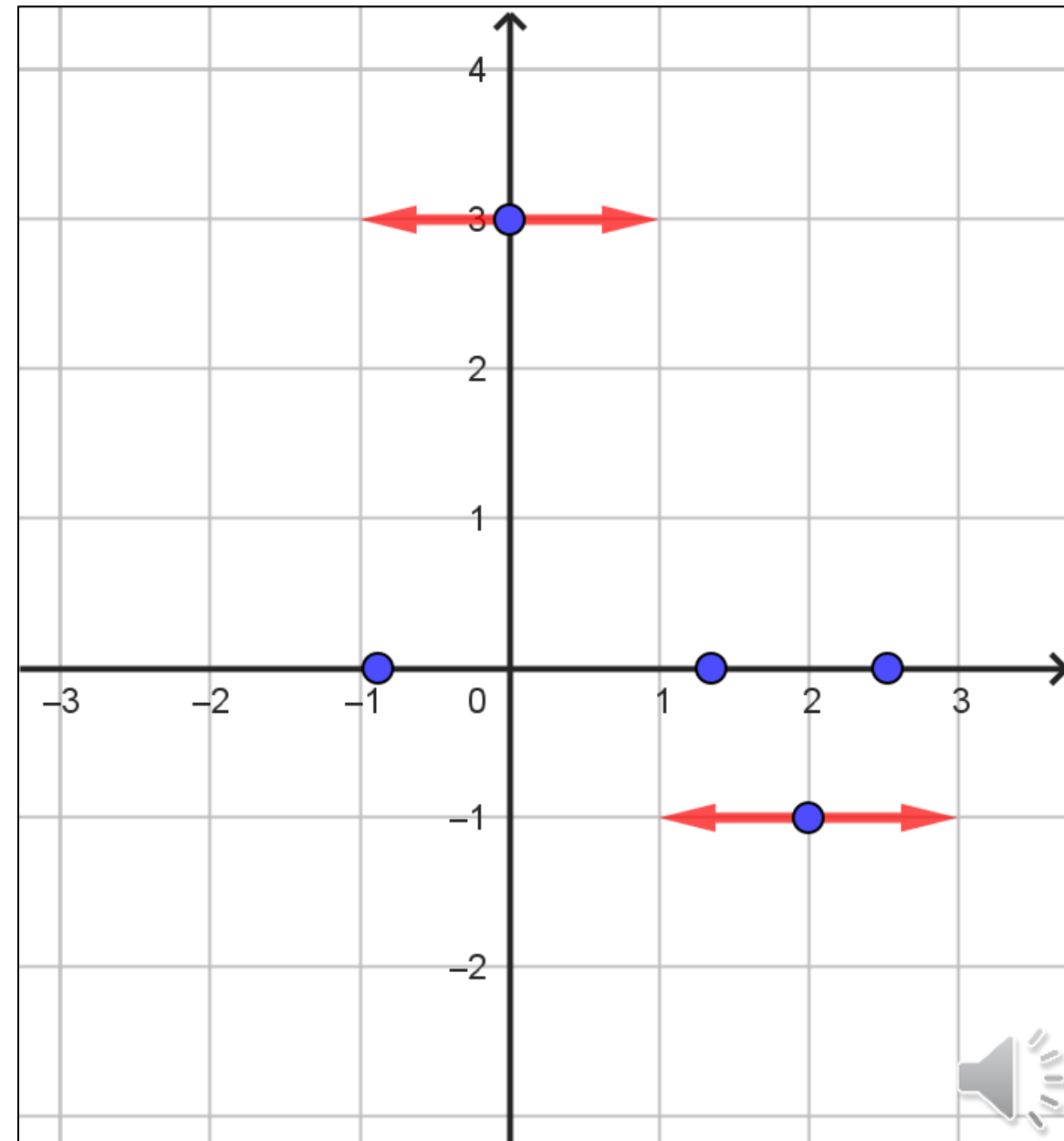
Example 1:

5. Plot the curve

$x$	$-\infty$	$0$	$2$	$+\infty$	
$f'(x)$	$+$	$0$	$-$	$0$	$+$
$f(x)$	$-\infty$	$3$	$-1$	$+\infty$	

Plotting the P.P.

$(0;3)$  ;  $(-0.9;0)$  ;  $(1.3;0)$  ;  $(2.5 ; 0)$



# How to plot the graph of a function?

Example 1:

5. Plot the curve

$x$	$-\infty$	$0$	$2$	$+\infty$
$f'(x)$	$+$	$\leftarrow 0 \rightarrow$	$\leftarrow 0 \rightarrow$	$+$
$f(x)$	$-\infty$	$3$	$-1$	$+\infty$

Start by the extrema.

Continue according to the table of variations

